The effectiveness of asset pricing models in the Brazilian market

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Abstract

Purpose
This study aimed to test the validity and effectiveness of three pricing models - CAPM, Fama-French Three-Factor Model, and Carhart Four-Factor Model - in the pricing of companies' shares listed on B3, the official Brazilian stock exchange. Apart from CAPM, the other models, which are multifactorial and evolutions of the first one, have not been widely tested in emerging markets.
Design/methodology/approach
The article used linear regression modeling. We use data from the Brazilian stock market. The sample spanned 10 years, between 2009 and 2020. The interval corresponds to the period between the Subprime and COVID-19 crises.
Findings
The results showed that the models were valid and effective in stock pricing. Carhart's four-factor model performed best among the three. The consistency in the performance of pricing models was greater than that shown by previous studies. The consistency in the performance of pricing models was greater than that shown by previous studies.
Originality
Although the three pricing models are well known and tested in developed markets, the empirical literature on emerging and Latin American countries is insufficient and limited. Pricing models have not been widely tested in emerging markets such as Brazil.

Key words: Asset Pricing; Fama-French Model; Carhart Model; CAPM.
JEL: G12, G11, G13, G17, C50, C58

1. Introduction

The most well-known and widespread pricing model worldwide is the capital asset pricing model (CAPM), developed almost jointly by Sharpe (1964) and Lintner (1965). After the CAPM, which is a linear one-factor model based on the concept of equity risk premium, other models emerged that are potentially more robust because they are multifactorial, although the original model has been corroborated by very relevant studies, such as the one by Black, Jensen and Scholes (1972). After being criticized, such as in the study by Fama and French (1993), the CAPM was a precursor to other pricing models. Among them, the Arbitrage Pricing Theory (APT) by Ross (1976), the Fama-French Three-Factor Model, presented in Fama and French (1993), and the Carhart Four-Factor Model, proposed by Carhart, can be mentioned. (1997).

According to Ross (1976), the APT was one of the first models suggested as an alternative to the traditional CAPM and one of the first models that suggested the use of multiple factors. Fama and French's Three-Factor Model (1993) adds two factors to the CAPM model: the size premium factor (SMB, or Small Minus Big factor), which represents the return that smaller companies give in relation to larger ones. And the book-to-market premium factor (HML, or High Minus Low factor), which takes into account the premium paid by companies with a high book value ratio over their market value. The addition of these two factors provided a variant model of the CAPM with greater explanatory power for the behavior of asset prices in the US market.

The Carhart Four Factor Model, also known as the Fama-French-Carhart Four Factor Model, was formulated by Carhart (1997). Based on the study by Jegadeesh and Titman (1993), which pointed to the existence of a premium based on the premise that stocks that had positive returns in the last year would have abnormally positive returns in subsequent years, Carhart considered the inclusion of a moment premium (WML, or the Winner Minus Loser factor) in the pricing model. The WML factor can be described as the tendency of stock prices to continue rising if they are in the process of reaching recent highs or to continue falling if they are in the process of reaching recent lows.

The mentioned models have been extensively tested over the last few decades, but mainly in developed-country markets. In emerging countries, literature is scarce and concentrated in countries in Europe (Zaremba & Czapkiewicz, 2017), Asia (Lin, 2017; Mosoeu and Kodongo, 2020; Singh, Singh and Prakash, 2022; Foye and Valentinčič, 2020) and Africa (Mosoeu and Kodongo, 2020) and Oceania (Mosoeu and Kodongo, 2020; Huynh, 2017; Nartea, Ward and Djajadikerta, 2009). Latin American markets are investigated in Foye (2018) and in the study by Hanauer and Lauterbach (2019), which analyzes 28 countries around the world. When there are large studies where several markets are investigated, Latin American countries are usually not analyzed (see Fama, French, 2017). This has an important practical implication for investment activities (fund management, portfolio management, analyst activities, asset management, company valuation, strategic management, etc.), considering the dynamism of emerging markets and the intense changes in the economic, political, and legal conditions of this category of countries. It should also be considered that the largest emerging markets in the world have expanded their political and economic relevance in the last two decades.

In addition to that, the scientific literature on valuation models is very scarce within the period between the world crises of 2008 - the subprime crisis - and the crisis that started in 2020 - the COVID-19 crisis. This was a period of very intense changes in the capital markets, especially in Brazil. This is why the following questions arise: (i) Could the results of past studies, which analyzed the market before these changes, be describing an analysis whose results are no longer adjusted or valid today? (ii) Do the valuation models derived from the CAPM present consistent results when applied to the Brazilian stock market, one of the most relevant emerging markets?

The main changes in the Brazilian capital market were the significant increase in the number of individual investors since 2011 and the increase in the volume of investments on the stock exchange. The number of individual investors grew by 453% from the beginning of 2011 to the end of 2020 and by 348% in their investments in the stock exchange in the same period, as pointed out by Ferreira and Ungaretti (2021). These changes have increased the liquidity and volume of the Brazilian market, as well as the number of available assets and, therefore, the sample used in past studies.

Therefore, this research aimed to test the validity and effectiveness of three asset pricing models—the CAPM, the Carhart Four Factor Model, and the Fama-French Three Factor Model—in the pricing of shares of companies listed on B3, the Brazilian stock exchange, especially in this new context of strong expansion of the Brazilian capital market, in addition to comparing the explanatory power of the previously mentioned models.

The paper is structured as follows: Section 2 describes the literature, while Section 3 discusses the data and methodology. Section 4 presents the regression results, and Section 4 concludes the article.

2. Literature Review

According to Tavares (2011), the CAPM model has been extensively tested in the Brazilian market, with 43 articles published between 1997 and 2008, in which 70% of the studies that analyzed the model consider it effective in pricing.

For the Fama-French Three-Factor and Carhart Four-Factor models, the situation is quite different in Brazil and other emerging markets, where the literature is relatively scarce and most of the studies that were carried out were on the Fama-French model, with a very small number of studies that approached the Carhart Four-Factor model. In emerging markets, studies by Karasneh and Almwalla (2011) indicate that the Fama-French Three-Factor Model is superior to the CAPM in terms of explanatory power, according to tests carried out in the markets of Australia, South Korea, Hong Kong, and Thailand, among other emerging ones.

In the Brazilian market, the conclusions are similar to those obtained with studies in other markets. However, the literature presents some divergences due to the diversity of periods analyzed and the constant changes in the market context. The main differences are found in the sign of the factors' return (influence).

Among the studies reviewed in the Brazilian market, Málaga and Securato (2004), Machado and Medeiros (2011), Mussa, Fama and Santos (2012), Bortoluzzo et al. (2016), and Garcia (2019) found the performance of the Carhart four-factor model to be superior to the Fama-French three-factor model and the latter superior to the CAPM in terms of explanatory power. Only Rizzi (2012) found the Fama-French model to perform better than the Carhart model, which in turn found it better than the CAPM.

Málaga and Securato (2004), who studied data from 1995 to 2003; Machado and Medeiros (2011), who analyzed data from 1995 to 2008; and Mussa, Fama, and Santos (2012), who studied data from 1995 to 2006, found a negative mean value for the SMB factor, different from that studied by Fama and French (1993) in the US market, in which the factor was positive. In addition, Rizzi (2012), who observed the period from 1995 to 2011, stated that this factor was not statistically significant in the Brazilian market. However, Laes and Silva (2014), who investigated data from 2002 to 2012; Bortoluzzo et al. (2016), who analyzed data from 2002 to 2013; and Garcia (2019), who investigated the years from 2010 to 2019, found the SMB factor with positive values.

For the HML factor, Machado and Medeiros (2011) and Bortoluzzo et al. (2016) found negative values, differing from the results obtained in the US market by Fama and French (1993) and Carhart (1997). Bortoluzzo et al. (2016) believe that the difference is due to the fact that, in the US, this factor brings together high-growth companies, for which the book value of assets is very small when compared to the market value. This type of company includes technology firms that have a large number of unaccounted-for intangible assets. In Brazil, companies that give this extra return would be those that have a history of consistent appreciation over the years, so that the market value ends up exceeding the book value, forming part of the low segment. Nevertheless, Málaga and Securato (2004), Mussa, Fama and Santos (2012), Rizzi (2012), Laes and Silva (2014), and Garcia (2019) found the factor to have positive values.

Finally, for the WML momentum factor, Machado and Medeiros (2011), Laes and Silva (2014), Bortoluzzo et al. (2016), and Garcia (2019) found positive values, in line with the results of Carhart (1997) in the US. Despite that, Mussa, Fama and Santos (2012) and Rizzi (2012) found negative values for this same factor. It is important to mention that Rizzi (2012) found values for the factor that don't have statistical significance, although the study attributes the result to certain limitations of the data, such as the liquidity levels of the shares of some companies included in the portfolios, in addition to the period chosen being very extensive for the analysis.

3. Method

This work follows the methodology suggested by Garcia (2019), which meets almost all of the steps adopted by Fama and French (1993). However, from the studies by Bortoluzzo et al. (2016), Rizzi (2012), Machado and Medeiros (2011), and Chague and De-Losso (2008), some methodological changes were adopted, which are presented in this section.

3.1 Sampling

The data explored in this research corresponds to the historical series of quotations of the shares of companies listed on the Brazil Stock Exchange and Over-the-Counter Market (B3). The selected companies and their shares met the following criteria: (i) shares with monthly closing prices from January 2009 to February 2020 in the analysis period. To avoid asymmetries caused by the subprime crisis in 2008, the year 2009 is used solely as a reference for calculating 2010 annual returns; (ii) companies that have available for consultation the accounting and financial information of the period of analysis; (iii) shares whose monthly prices fluctuate less than 100%; (iv) companies that are not in the financial sector; (v) shares, whose calculated liquidity according to the Negotiability Index (NI) of the stock exchange, according to B3 (2018), adds up to 99.5% in descending order of NI, as is done with the iBrA index, according to B3 (2020), adding 0.5% more to expand the sample; (vi) for listed companies that have more than one type of share, only the most liquid class is considered.

3.2 Building Portfolios

Given the large number of characteristic changes of an emerging market such as the Brazilian one, there are companies that have had an abrupt drop or increase in liquidity, which would suddenly include or eliminate them from the sample. Thus, to standardize and capture as many companies as possible, which would allow us to make better statistics, an Excel interpolation of the quotation data was performed with a tolerance of up to two months, as recommended by Chague and De-Losso (2008).

12 portfolios were created, month by month, according to the intersection of the factors of:

Market value (MV)—obtained by multiplying the closing price of the trading session by the number of shares in the company. The companies were divided into two groups: big and small, with half of the companies with the smallest MV in the small group and the other half with the largest MV in the big group. The market value is calculated with Equation 5:

$$VM_{i,t} = P_{i,t} \times N_{i,t} \qquad \text{[Equação 3]}$$

where *VM* is the market value of company i in month t; *P* is the monthly closing price of shares of company i in month t; and *N* is the number of shares of company i in month t.

The book-to-market ratio (B/M) is the ratio obtained between the book value and the market value (MV) of the company. Based on this ratio, the shares were divided into three groups: low, medium, and high, each with a third of the shares, in which the low group will be made up of the third of the shares of the companies with the lowest B/M, the medium with the middle percentile of B/M, and the high group with the highest fraction. The ratio was calculated using Equation 4:

$$B/M_{i,t} = \frac{VPA_{i,\,dez(t-1)} x N_{i,dez(t-1)}}{VM_{i,\ t}} \qquad \text{[Equação 4]}$$

where $BV_{i,\, dec(t-1)}$ is the book value per share of company i calculated in December of the immediately preceding year; $N_{i,dec(t-1)}$ is the number of shares of company i calculated in December of the previous year; $VM_{i,\, t}$ is the market value of the company. The multiplication of $BV_{i,\, dec(t-1)}$ and $N_{i,dec(t-1)}$ refers to the book value of the company, which was calculated in this way by the availability of data on the Economatica platform.

The momentum indicator (MI) was introduced by Carhart (1997), based on evidence from Jegadeesh and Titman (1993), who indicated a higher return for a strategy in which stocks with higher returns were purchased in the past, financed by the sale of stocks with low or negative returns over a period of 3 to 12 months.

This indicator was calculated based on the return on assets over the past twelve months, as explained by Glabadanidis (2009). They will be divided into two groups: winners and losers. The first group will be made up of half of the companies with the best return for the year, and the remaining 50% of the stocks with the worst return. The equation used to calculate the MI was Equation 5:

$$IM_{i,t} = \frac{P_{i,t}}{P_{i,\,t-12}} \qquad \text{[Equação 5]}$$

where $MI_{i,t}$ is the momentum indicator of stock i in month t; $P_{i,t}$ is the price of stock i in month t; $P_{i,\,t-12}$ is the price of stock i in the last year.

The twelve portfolios shown in Table 1 were obtained using the same procedure adopted by Garcia (2019), as follows:

The portfolios will be modified each month according to the variations of the indicators. It is important to highlight that the methodology used by Garcia (2019) for the construction of portfolios, which is the same as that used by Fama and French (1993), results in an important asymmetry among the numbers of assets that make up each portfolio, especially in cases of less developed markets because of the lack of liquidity of several stocks. As a result, some portfolios are composed of a lot fewer assets than others. Since the present study considered the premises of Blume and Friend (1973), which point to the reduction of statistical errors with the use of portfolios with several assets, the procedure suggested by Bortoluzzo et al. (2016), in which stocks are first ordered by their B/M and then grouped by MV numbers and returns, was adopted here. This procedure is the opposite of that adopted by Fama and French (1993), who organized the portfolios by observing the shares first by the MV and then by their B/M. The subdivisions of each group would remain the same as in the previous methodology, only changing the order of selection. The exposed logic is illustrated in Figure 1, in which the total number of companies is first divided into three groups according to their B/M ratio, then each of these groups is again subdivided by its size, doubling the number of asset groups, and, finally, each of the six groups is subdivided, but this time by their return, totaling 12 groups of companies. With this method, the portfolios result in approximately the same number of assets, avoiding asymmetries.

3.3 Calculation of returns

To calculate the portfolio returns, the weighting of the asset returns was made using the weighted average by the Negotiability Index (NI). This, in order to standardize the study, since the market return used is the return of the theoretical portfolio of Ibovespa and the NI calculation follows the methodology of B3 (2018). The equation used to calculate the weighted annual return$R_{P,t}$, for a portfolio p with n assets, is Equation 6:

$$R_{P,t} = \sum_{i=0}^{n} \frac{IN_{i,t}}{IN_{P,t}} (R_{i,t}) \quad \text{[Equação 6]}$$

Where $R_{P,t}$ is the weighted annual return of portfolio p in month t; $NI_{i,t}$ is the negotiability index of asset i in month t; $NI_{P,t}$ is the total negotiability index of portfolio p in month t; $R_{i,t}$ is the annual return on asset i in month t.

3.4 Premium Calculation

There are four premiums calculated: Market premium (P_{MKT_t}); Size factor premium (SMB_t); Premium for the book-to-market factor (HML_t); and Momentum factor premium (WML_t). They were calculated as indicated below.

a) Market Premium (P_{MKT_t}): It arises from the difference between the returns earned by a market portfolio R_{M_t}, defined in this study as the theoretical portfolio of Ibovespa, according to B3 (2018), and the returns of an asset linked to the risk-free rate R_{f_t}, defined as the Selic rate, which is the Brazilian Central Bank's basic interest rate. The same indices were used by Chague and De-Losso (2008). Equation 9 determines the market premium:

$$P_{MKT_t} = R_{M_t} - R_{f_t} \quad \text{[Equação 7]}$$

Where P_{MKT_t} is the market premium in month t; $R_{M_t}$is the market portfolio return in month t; R_{f_t} is the risk-free rate return in month t.

b) Size factor Premium (P_{SMB_t}): The expression that represents the calculation of the size factor is Equation 8:

$$P_{SMB_t} = \overline{R}_{S,t} - \overline{R}_{B,t} \quad \text{[Equação 8]}$$

Where $P_{SMB_t}$is the premium for the size factor in month t;

$$\overline{R}_{S,t} = \frac{R_{H\,/\,S/Los} + R_{H\,/\,S/Win} + R_{M\,/\,S/Los} + R_{M\,/\,S/Win} + R_{L\,/\,S/Los} + R_{L\,/\,S/Win}}{6}$$

$$\overline{R}_{B,t} = \frac{R_{H\,/\,B/Los} + R_{H\,/\,B/Win} + R_{M\,/\,B/Los} + R_{M\,/\,B/Win} + R_{L\,/\,B/Los} + R_{L\,/\,B/Win}}{6}$$

Observe that $\overline{R}_{S,t}$ and $\overline{R}_{B,t}$, are the averages of portfolio returns denoted as Big and Small. And that the variables that describe them follow the notation of Table 1.

c) Premium for the book-to-market factor (P_{HML_t}): it is given by the difference between the average returns of portfolios with high book-to-market index $\overline{R}_{H,t}$, with the average return of portfolios with low index $\overline{R}_{L,t}$, according to Equation 9:

$$P_{HML_t} = \overline{R}_{H,t} - \overline{R}_{L,t} \quad \text{[Equação 9]}$$

Where:

P_{HML_t} is the premium for the book-to-market factor in month t;

$$\overline{R}_{H,t} = \frac{R_{H\,/\,S/Los} + R_{H\,/\,S/Win} + R_{H\,/\,B/Los} + R_{H\,/\,B/Win}}{4}$$

$$\overline{R}_{L,t} = \frac{R_{L\,/\,S/Los} + R_{L\,/\,S/Win} + R_{L\,/\,B/Los} + R_{L\,/\,B/Win}}{4}$$

d) Momentum factor Premium (P_{WML_t}): is given by the difference between the average returns of the portfolios with higher returns in year $\overline{R}_{W,t}$, with the average return of the portfolios with lower returns $\overline{R}_{L,t}$, according to Equation 10:

$$P_{WML_t} = \overline{R}_{W,t} - \overline{R}_{L,t} \qquad \text{[Equação 10]}$$

Where:
P_{WML_t} is the Premium for the momentum factor in month t;

$$\overline{R}_{W,t} = \frac{R_{H\,/\,B/Win} + R_{H\,/\,S/Win} + R_{M\,/\,B/Win} + R_{M\,/\,S/Win} + R_{L\,/\,B/Win} + R_{L\,/\,S/Win}}{6}$$

$$\overline{R}_{L,t} = \frac{R_{H\,/\,B/Los} + R_{H\,/\,S/Los} + R_{M\,/\,B/Los} + R_{M\,/\,S/Los} + R_{L\,/\,B/Los} + R_{L\,/\,S/Los}}{6}$$

3.4 Models applied

After calculating the factors and premiums for the construction of the portfolios and calculating the premiums, statistical methods were applied to quantify the correlation between the models and their explanatory power. The models applied were slightly modified from those presented and were formulated according to Equations 11, 12 and 13 to perform the regressions:

CAPM:
$$RCi,t - Rft = a + b[Rmt - Rft\,] + ei,t \qquad \text{[Equação 11]}$$

Modelo de Três Fatores de Fama–French:
$$RCi,t - Rft = a + b[Rmt - Rft\,] + s[P_{SMB_t}\,] + h[P_{HML_t}\,] + ei,t \qquad \text{[Equação 12]}$$

Modelo de Quatro Fatores de Carhart:
$$RCi,t - Rft = a + b[Rmt - Rft\,] + s[P_{SMB_t}] + h[P_{HML_t}\,] + w[P_{WML_t}\,] + ei,t \qquad \text{[Equação 13]}$$

Where RCi,t is the return on portfolio i in month t; Rft is the return on risk-free asset in month t; Rmt is the return on the market portfolio in month t; P_{SMB_t} is the prize for the size factor in month t; P_{HML_t} is the premium for the book-to-market factor in month t; P_{WML_t} is the premium for the momentum factor in month t; ei,t is the residual of the model referring to the return of portfolio i in month t; a, b, s, h and w are the parameters of the temporal linear regression.

4. Results

In this section, the descriptive statistics of the portfolios, the analysis of the independent variables, and the analysis of the models are presented.

4.1 Descriptive statistics of the portfolios

The number of assets analyzed varied from month to month, following the evolution of the Brazilian stock market. Thus, the number of companies analyzed in December 2009, which complied with all the requirements mentioned in the methodology, was 105, increasing to a total of 151 companies in the last month of analysis, that is, January 2020.

Due to the change in methodology and the order of selection of assets suggested by Bortoluzzo et al. (2016), the amount of assets per portfolio for all portfolios was quite uniform, with an average of 10 assets for the entire period of analysis.

To assess the risk-return profile of the portfolios, the monthly returns and their standard deviations were calculated; the results are shown in Table 2.

It can be seen that the average monthly returns of the portfolios range from -3% to 5.9% for the period studied, with standard deviations of up to 6.7%. These results differ from the results of Garcia (2020) and Mussa, Fama and Santos (2013), who found more uniform returns in the ranges of -0.4% to 3% but with higher standard deviations, reaching up to 15%. This divergence may be due to the difference in the analyzed period, as in the case of the study by Mussa, Fama and Santos (2013), or the introduction of the liquidity filter and the change in methodology that standardized the amount of assets per portfolio. Thus, the descriptive results of the portfolios are more similar to those of Bortuluzzo et al. (2016), who obtained average monthly returns in the range of -4,6% to 5,5%. However, the average returns of all portfolios in this study, as in the studies already mentioned, were similar, remaining in the range of 1.2% to 1.6% of monthly return.

Observe that the highest positive return and the highest loss (negative return) were recorded for portfolios with Small-type assets, as well as higher standard deviations.

It is interesting to highlight the similarity of the standard deviation value of 6.7% associated with the maximum return of 4.5% of the US market, calculated by Fama and French (1993), with the values of maximum standard deviation (6.67%) associated with the maximum return (5.84%) shown in Table 1. However, it is not possible to assume a similar volatility between the Brazilian market and the US market, as this may be mainly due to the change in methodology; even so, this may point to an evolution of the Brazilian market; unfortunately, the study by Bortoluzzo et al. (2016) does not present the standard deviations of the portfolio returns, making it impossible to compare this aspect of the results.

4.2 Analysis of independent variables (risk factors)

Table 3 presents the descriptive statistics of the risk factors analyzed in the period, that is, the average monthly return, standard deviations, and t-test statistics, associated with their p-values.

In general, all the factors showed lower standard deviations than other studies carried out in the Brazilian market. With regard to the market factor, an average return of -0.1% was found. A positive return was expected, as verified in several studies of the Brazilian market. It should be observed, however, that the study by Garcia (2019) found an average monthly return of 0% for the market factor, a value very close to the one obtained here.

The SMB factor was positive, as expected, converging with the results of the studies by Fama and French (1993) and Garcia (2019). Despite this, the statistical significance of this factor, as well as that of the market factor, is reduced, since the p-value is well above 0,05, which at first would confirm the analysis by Málaga and Securato (2004) and that of Mussa, Fama and Santos (2013). Even so, it is not possible to say that market and SMB factors are not significant until the time of performing the regressions, since this statistic is based on the average and not on each of the periods, which may be a biased result.

The value associated with the HML factor was negative, confirming the results of Bortoluzzo et al. (2016) but differing from those of Fama and French (1993), in the US market, and Garcia (2019), in the Brazilian market.

Finally, the value associated with the momentum factor was positive, as expected, coinciding with the results of studies by Garcia (2019) and Bortoluzzo et al. (2016), in addition to the study by Carhart (1997), in the US market.

It is important to mention that the results of the premiums may have been affected because of the adjustments in the methodology, with the exception of the market premium, which is independent of the construction of portfolios.

A correlation test between the factors was also performed to prove their orthogonality. The results are shown in Table 4.

Most factors showed a low correlation, confirming the orthogonality. However, the size factor and the market factor had a relatively high correlation with the momentum factor, a fact that is intriguing because we have not seen similar results in any other study. The literature shows correlation values that hardly exceed 0.5 per module.

4.3 Analysis of the explanatory power of models

The temporal regression was performed according to Equation 11 for the CAPM model, Equation 12 for the Fama-French three-factor model, and Equation 13 for the Carhart four-factor model. All temporal regressions were performed using the least-squares method. The regression coefficient "a," which is the only one that is not associated with any factor, can be interpreted as the model residual, that is, a parameter that, if not set to zero, has to be explained by factors other than those considered on each model.

For the interpretation of p-values, it is important to mention that the hypothesis formulated for their calculation was whether the parameters a, b, s, h, and w were different from zero. Therefore, what was expected is that the ideal p-values for b, s, h, and w would be close to zero, which implies the significance of the factor in explaining the portfolio returns. Parameter a, on the other hand, is expected to have a high p-value, which would indicate that the model had a small residual, implying that the factors used explain almost all of the portfolio returns.

The Durbin-Watson Test, which interprets the autocorrelation of the points, that is, the similarity between them as a function of the time differential between them, is a scale from 0 to 4, in which values close to 2 indicate low autocorrelation, values close to 4 show negative autocorrelation, and values close to zero indicate positive autocorrelation. However, these values are approximations, and in order to have a more accurate notion of the real critical values (dL and dU), the critical values were taken as a reference as a function of the number of parameters k, the number of factors of each model, for $n = 123$. As the critical value for n exactly equal to 123 is not tabulated, a linear interpolation between $n = 150$ and $n = 100$ was performed. For this test, the results were expected to be lower than the dL value.

The CAPM model was the only one in which "Return vs. Factor" graphs were constructed, given that it is the only model with a number of dimensions lower than three. The charts are presented in book-to-market index order, that is, the charts of the high portfolios are shown first, then those of the medium category, and finally the low ones.

It is possible to observe that the graphs of the high portfolios, especially the HSWin, have points that are quite outside the regression. Its exclusion could be plausible. However, as no objective exclusion criterion was identified in the methodology for this type of occurrence, the divergent points were also considered in the calculation.

The graphs of the medium portfolios were less divergent from the regression, with a smaller standard deviation and a much more accurate adjustment, as can be seen in the numerical results of the regressions shown in Table 4.

In the same way as in the high portfolios, the low portfolios presented divergent points to the regression, especially the LSWin portfolio, but even so, they were considered for the calculation of the least squares of the regression. We can see in the graphs of Figures 2, 3, and 4 that there is a higher yield in the win portfolios, which are normally above 0%. This result was expected, given the arrangement and organization that are characteristic of these portfolios.

The numerical results of the regression with the CAPM model are shown in Table 5. The numbers show that there is a considerable coefficient of determination for most portfolios, except for the HSWin, LBLoss, and LSWin portfolios, which had R2 coefficients lower than 30%. The average of the R2 coefficients for this model was 48.2%.

Despite the reduced coefficient of determination for some portfolios, the p-value of b was close to zero in up to three significant figures, which gives relevant statistical significance to the market factor. However, the p-value of a was reduced in all portfolios, so the data suggest that the market factor is not sufficient to describe the portfolio returns.

Finally, the Durbin-Watson Test, in this case, provides the critical values of dL = 1,563 and dU = 1,597. As all test values for all portfolios are below dL, it can be said that the data are positively self-correlated.

For the Fama-French three-factor model, a considerable increase in the values of the R2 coefficient was found, with a substantial increase in explanatory power for all portfolios, as seen in Table 6. Only for the LBLos portfolio, R2 was below 50%. The increase in explanatory power was 43%, and the average R2 for this model was 68.9%.

Parameters b and s had p values that confirmed their statistical significance in all portfolios. Despite this, the h factor, although having significance in most portfolios, presented very high p values in four portfolios. In addition, the parameter a again had small p-values, suggesting that market, size, and book-to-market factors are insufficient to describe portfolio returns.

The Durbin Watson Test, in this case, had values of dL = 1.529 and dU = 1.632, values that place the results as positively correlated, as for the CAPM model.

For Carhart's four-factor model, a new increase in the values of the R2 coefficient was found, as shown in Table 7. The values of this coefficient were on average 4.4% higher in relation to the three-factor Fama-French model, with an average explanatory power of 72%. This being, then, the model with the greatest explanatory power among those tested in this study,

Furthermore, it is the only model among the three that presented the highest p-values for the parameter, although it was for only half of the portfolios. High values of the a parameter suggest that the model can explain the returns with the four factors that compose it. However, the p-values of the factors, with the exception of the market factor, were quite high. The size factor was not significant in three of the twelve portfolios, the book-to-market factor in seven of the twelve portfolios, and the momentum factor in five of the twelve portfolios, according to the data. It is important to remember that this may be due to the strong correlation between the market and size factors and the momentum factor, as indicated in Table 4, since the market factor was statistically significant in almost all portfolios and in all the models, and when it was not, the other factors were significant. Even so, it was not possible to establish a pattern between the p values and the characteristics of the portfolios.

Finally, the Durbin Watson test for this model had critical values of dL = 1.512 and dU = 1.650, indicating once again a positive autocorrelation of the data.

5. Discussion

5.1 Theoretical implications

The study has some theoretical implications. The academic literature in Brazil lacks studies on asset pricing. There are not many studies published in English that address the topic. In addition, the study contributes to the literature on emerging markets. The financial markets of developed economies are investigated further. The results showed that the four-factor models had superior results. The findings were similar to those in the international literature for developed economies.

5.2 Policy/Managerial implications

Regarding practical or managerial implications, the study analyzes different factorial models for the stock market in Brazil. The evidence contributes to the pricing of assets by decision-making agents, investment funds, and managers, among others. Active pricing models are used as a proxy for calculating the cost of capital in a company valuation. In addition, the findings serve to monitor the performance of investment funds. Investors and resource savers can choose funds that present higher returns than those priced by factorial models.

5.3 Limitations and future research agenda

The research was limited to testing three factorial models in the Brazilian stock market in periods that did not include global economic crises. For future research, we suggest investigating the models in periods of strong market instability. As was the case with the subprime crisis in 2008 and the COVID-19 outbreak in 2020. Research that addresses the most recent factorial pricing models (see Fama and French, 2015) is also suggested for investigation.

6. Conclusions

Three pricing models were tested in this work: the one-factor CAPM model and the multi-factor models of Fama-French and Carhart to price shares in the Brazilian capital market using data from 2009 to 2020, that is, between the Subprime Crisis and the COVID-19 Crisis. The objective was to verify whether the evolution of the Brazilian capital market, as expressed by the increase in the number of assets and liquidity, especially in the period from 2014 to 2020, had any impact on the explanatory power of the models.

The results obtained suggest that: (i) Carhart's Four Factor Model proved to be the most suitable for pricing assets in the Brazilian market, with an average R2 of 72%; (ii) the explanatory power of the three models, given by the R2 coefficient, was greater than that found in the studies by Mussa, Fama and Santos (2013), Bortoluzzo et al. (2016), Rizzi (2012), and Machado and Medeiros (2011). This suggests that the evolution of the Brazilian capital market may have promoted an improvement in the explanatory power of the three models analyzed. (iii) Carhart's Four Factor Model had the lowest residual among the models presented, which suggests that the model is able to explain the most relevant variables that affect excess returns in the Brazilian market; (iv) the market factor continues to be the most relevant, since in all models and for all portfolios, it presented statistical significance at the level of 5%. In spite of this, it did not prove to be a sufficient factor to explain the additional returns; (v) the methodology suggested by Bortoluzzo et al. (2012) actually promotes a better distribution of portfolios and statistically more consistent results; (vi) the high correlations found in this study between the market and size factors and the momentum factor may have produced the moderately positive autocorrelation detected in the Durbin Watson Test. This autocorrelation can cause a distortion of the values obtained with the t test and, consequently, of the p-values.

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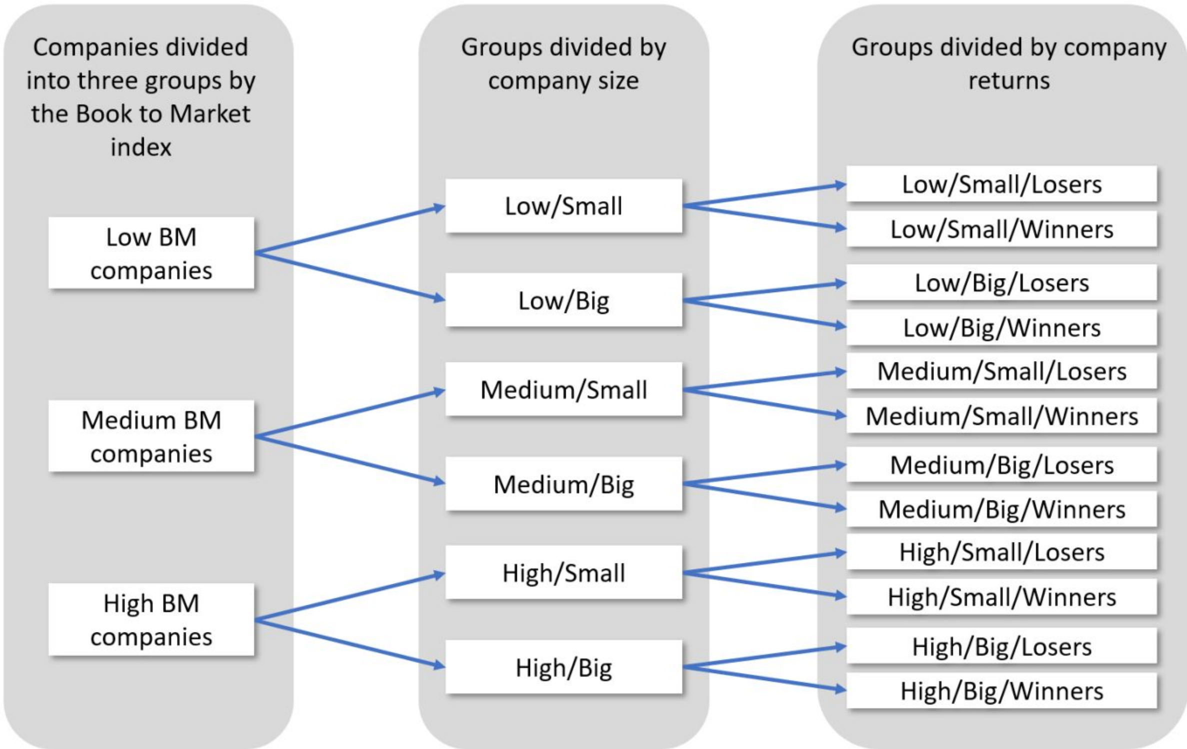

Figure 1. Portfolio separation for emerging markets methodology
Source: Bortoluzzo et al. (2016)
Note: The figure shows the composition of the analyzed portfolios, observing the shares first by MV and then by their B/M and the subdivisions of each group.

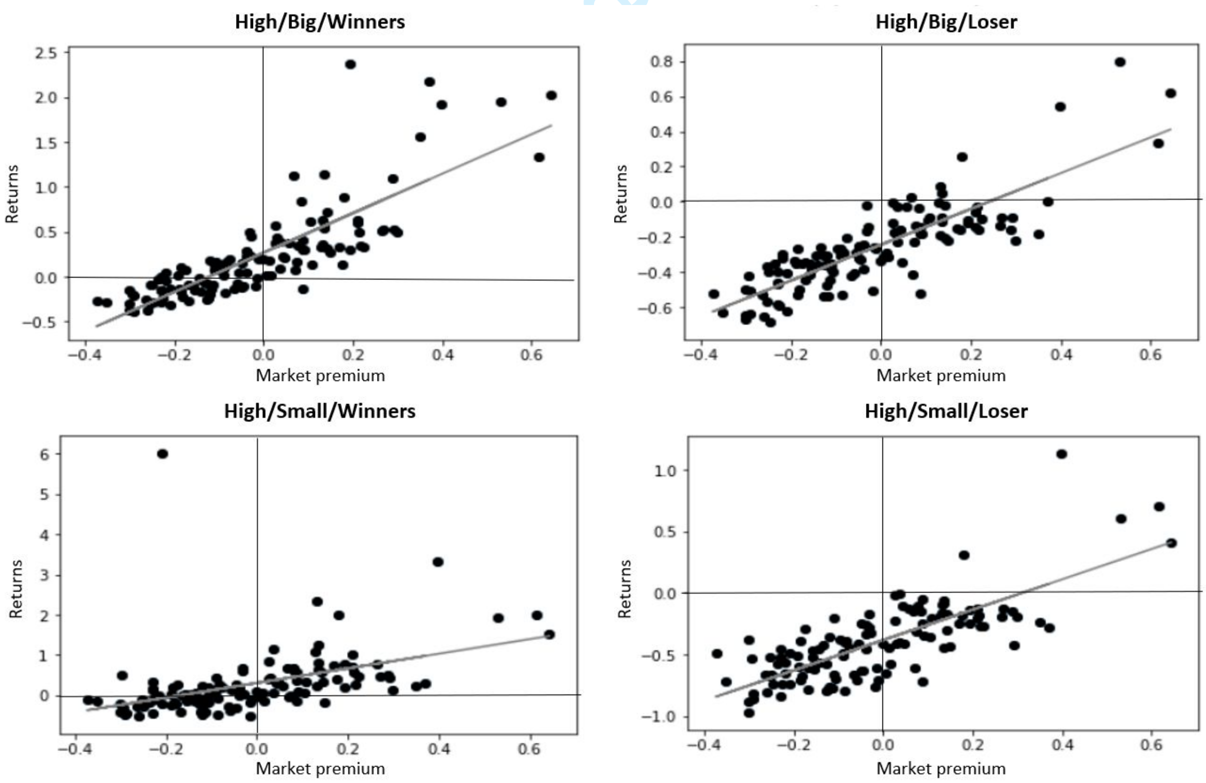

Figure 2. CAPM model adjustment graphs for *high book-to-market* portfolios

Note: Figure shows the linear relationship between the market premium and the returns of the adjusted CAPM models for the High book-to-market portfolios

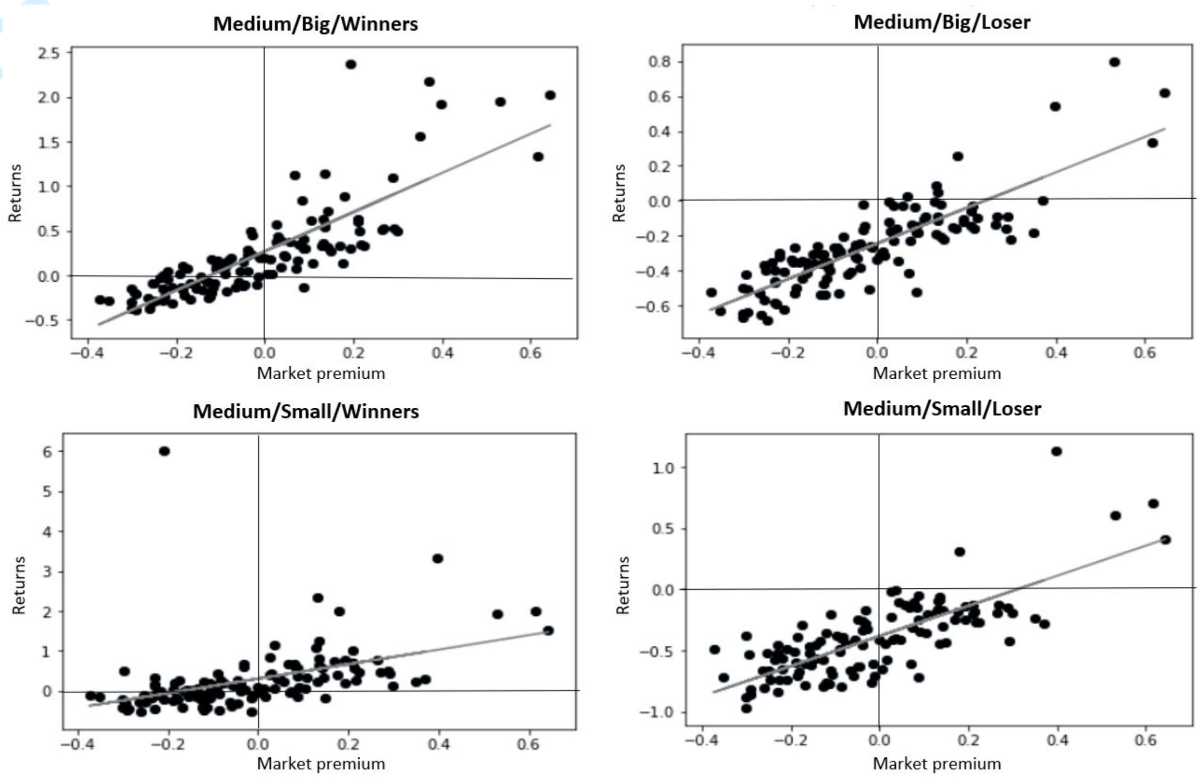

Figure 3. CAPM Model Adjustment for *medium book-to-market* portfolios

Note: Figure shows the linear relationship between the market premium and the returns of the adjusted CAPM models for the Medium book-to-market portfolios

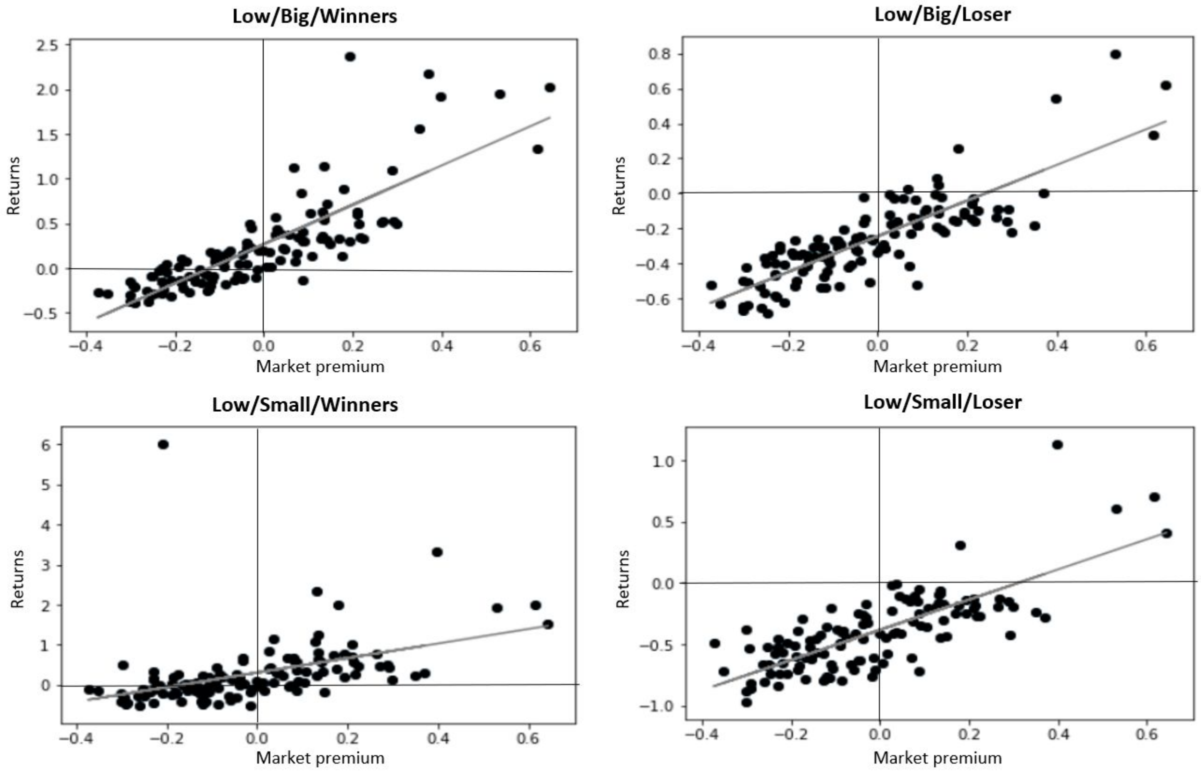

Figure 4. CAPM Model Adjustment for *low book-to-market* portfolios

Note: Figure shows the linear relationship between the market premium and the returns of the adjusted CAPM models for the Low book-to-market portfolios

Table 1. Portfolios built based on the MV, B/M and MI Indicators

Portfolio	Composition	Portfolio	Composition
High/Big/Winners	Stocks with High BM, high MV and with favorable past returns	Medium/Small/Winners	Medium BM stocks, low MV and with favorable past returns
High/Big/Losers	Stocks with High BM, high MV and with unfavorable past returns	Medium/Small/Losers	Stocks with Medium BM, low MV and with unfavorable past returns
High/Small/ Winners	Stocks with high BM, low MV and with favorable past returns	Low/Big/Winners	Stocks with low BM, high MV and with favorable past returns
High/Small/Losers	Stocks with High BM, low MV and with unfavorable past returns	Low/Big/Losers	Stocks with low BM, high MV and with unfavorable past returns
Medium/Big/Winners	Stocks with medium BM, high MV and with favorable past returns	Low/Small/Winners	Stocks with low BM, low MV and with favorable past returns
Medium/Big/Losers	Stocks with medium BM, high MV and with unfavorable past returns	Low/Small/Losers	Stocks with low BM, low MV and with unfavorable past returns

Note: The table shows the distribution of the analyzed portfolios.

Table 2. Average monthly returns and standard deviations of portfolios

Portfolios	Average monthly returns (2010 -2020)	Standard Deviations
HBWin	2.39%	3.61%
HBLos	-1.47%	1.77%
HSWin	2.67%	4.96%
HSLos	-2.97%	2.30%
MBWin	3.18%	2.05%
MBLos	-0.24%	1.23%
MSWin	3.55%	3.15%
MSLos	-0.48%	1.76%
LBWin	3.91%	2.33%
LBLos	0.60%	1.40%
LSWin	5.84%	6.67%
LSLos	-0.06%	2.27%

Note: The table shows the values of the average returns of the portfolios between the years 2010 and 2020. The values of the standard deviation are also presented.

Table 3. Average monthly premium and statistical significance

Factors	Average monthly	Standard Deviations	T test	p-value
Market	-0.001	0.015	-0.980	0.329
SMB (Size)	0.003	0.025	1.154	0.251
HML (Book-to-Market)	-0.037	0.028	-10.417	0.000
WML (Momentum)	0.041	0.022	23.609	0.000

Note: The table shows the values of the monthly mean, standard deviation, t test and p-value of the factors analyzed

Table 4. Correlation matrix between factors

Factors	Market	*SMB* (Size)	*HML* (*Book-to-Market*)	*WinMLos* (Momentum)
Market	1.0000	-	-	-
SMB (Size)	0.3302	1.0000	-	-
HML (Book-to-Market)	0.0366	-0.4627	1.0000	-
WinMLos (Momentum)	0.6312	0.7161	-0.2224	1.0000

Note: The table shows the correlations between the Market, Size, Book-to-Market and Momentum factors

Table 5. Results of the regression with the CAPM model

Portfolios	R^2	a	p-value	b	p-value	Durbin-Watson
HBWin	0.676	0.266	0.000	2.199	0.000	0.978
HBLos	0.698	- 0.245	0.000	1.018	0.000	0.891
HSWin	0.206	0.304	0.000	1.813	0.000	1.293
HSLos	0.585	-0.382	0.000	1.2317	0.000	0.783
MBWin	0.621	0.3754	0.000	1.119	0.000	1.025
MBLos	0.591	-0.117	0.000	0.624	0.000	1.088
MSWin	0.538	0.449	0.000	1.691	0.000	0.513
MSLos	0.417	-0.142	0.000	0.776	0.000	0.455
LBWin	0.415	0.503	0.000	1.069	0.000	0.728
LBLos	0.280	-0.016	0.264	0.506	0.000	0.264
LSWin	0.249	0.929	0.000	2.951	0.000	0.553
LSLos	0.509	-0.087	0.000	1.127	0.000	0.460

Note: The table shows the linear regression results of the CAPM model for the analyzed portfolios.

Table 6. Results of the regression with the Fama-French model

Portfolios	R^2	a	p-value	b	p-value	S	p-value	h	p-value	Durbin Watson
HBWin	0.701	0.328	0.000	2.031	0.000	0.269	0.006	0.207	0.010	1.108
HBLos	0.790	-0.258	0.000	0.893	0.000	0.220	0.000	-0.009	0.762	1.346
HSWin	0.830	0.630	0.000	0.466	0.005	2.210	0.000	1.170	0.000	0.865
HSLos	0.783	-0.389	0.000	0.967	0.000	0.459	0.000	0.041	0.372	1.359
MBWin	0.653	0.378	0.000	1.019	0.000	0.172	0.002	0.029	0.511	1.124
MBLos	0.656	-0.124	0.000	0.554	0.000	0.123	0.000	-0.005	0.853	1.414
MSWin	0.665	0.457	0.000	1.371	0.000	0.548	0.000	0.090	0.215	0.856

MSLos	0.583	-0.132	0.000	0.580	0.000	0.334	0.000	0.071	0.094	0.794
LBWin	0.534	0.421	0.000	0.996	0.000	0.153	0.039	-0.206	0.001	0.781
LBLos	0.479	-0.074	0.000	0.441	0.000	0.131	0.004	-0.140	0.004	0.447
LSWin	0.862	0.462	0.000	1.882	0.000	2.001	0.000	-1.028	0.000	0.726
LSLos	0.734	-0.161	0.000	0.950	0.000	0.329	0.000	-0.162	0.000	0.913

Note: The table shows the linear regression results of the Fama-French model for the analyzed portfolios.

Table 7. Results of the regression with the Carhart model

Portfolio	R^2	a	p-value	b	p-value	s	p-value	h	p-value	w	p-value	DW
HBWin	0.830	-0.655	0.000	1.160	0.000	-0.700	0.000	0.085	0.166	1.547	0.000	0.983
HBLos	0.797	-0.367	0.000	0.797	0.000	0.113	0.072	-0.023	0.460	0.171	0.038	1.302
HSWin	0.860	-0.082	0.575	-0.165	0.400	1.509	0.000	1.082	0.000	1.120	0.000	0.778
HSLos	0.785	-0.472	0.000	0.894	0.000	0.378	0.000	0.026	0.529	0.130	0.243	1.329
MBWin	0.707	0.041	0.589	0.720	0.000	-0.160	0.069	-0.012	0.781	0.531	0.000	0.994
MBLos	0.656	-0.132	0.005	0.547	0.000	0.116	0.034	-0.006	0.829	0.012	0.868	1.404
MSWin	0.674	0.243	0.062	1.182	0.000	0.337	0.026	0.063	0.387	0.337	0.087	0.810
MSLos	0.585	-0.756	0.320	0.630	0.000	0.389	0.000	0.078	0.073	-0.088	0.446	0.819
LBWin	0.594	0.005	0.960	0.628	0.000	-0.257	0.035	-0.258	0.000	0.654	0.000	0.590
LBLos	0.499	-0.212	0.002	0.318	0.000	-0.006	0.942	-0.157	0.000	0.218	0.032	0.404
LSWin	0.916	-0.947	0.000	0.634	0.005	0.614	0.002	-1.202	0.000	2.215	0.000	0.589
LSLos	0.734	-0.136	0.089	0.973	0.000	0.353	0.000	-0.016	0.001	-0.039	0.746	0.920

Note: The table shows the linear regression results of the Carhart model for the analyzed portfolios.